

Coulomb induced diffraction of energetic hadrons into jets

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Abstract

The electromagnetic (e.m.) current conservation and renormalizability of QCD are used to calculate the amplitude of energetic hadron(photon) diffraction into several jets with large relative transverse momenta off the nucleon(nucleus) Coulomb field. Numerical estimates of the ratio of e.m. and strong amplitudes show that within the kinematic range where the leading twist approximation for the strong amplitude is applicable, the e.m. contribution can be neglected. In pA scattering at LHC and in the fragmentation of a photon into two jets in ultraperipheral AA collisions in the black limit (which maybe realistic at LHC) e.m. contribution may win.

1 Introduction

Diffraction of hadrons into jets off the gluon field of a nucleon(nucleus) is becoming an effective method of investigation of the light cone wave functions of hadrons. The H1 and ZEUS experiments at HERA discovered processes of hard diffractive electroproduction of vector mesons off proton target [1]

with the striking properties predicted in [2, 3, 4] based on the new QCD factorization theorems for high energy exclusive processes. The color transparency phenomenon in the coherent pion dissociation into two high p_t jets off nuclear target has been predicted in [2] and observed at FNAL [5] including a very strong ($\propto A^{1.55}$) dependence of the cross section on the atomic number. Moreover the measured dependence of the differential cross section on transverse and longitudinal components of the jet momentum agrees well with the one which is characteristic for high momentum tail of pion wave function evaluated in [6]. A potential background for the hard QCD physics which has similar dependence on the atomic number $\propto Z^2/A^{1/3}$ arises from the inelastic diffraction of a projectile pion off the Coulomb field of the target. Similar background exists for the process of the photon diffraction into two jets which is currently being studied by ZEUS(HERA)[7]. It is also present for the process of diffractive proton fragmentation into three jets which is one of the promising new hard processes for studies at RHIC, LHC and Tevatron [8]. Processes of high energy hadron scattering off the Coulomb field are interesting by themselves because their amplitudes are rapidly increasing with the energy of the collision as a result of the zero mass of the photon and the decrease with the incident energy of the minimum momentum transfer to the nuclear target, t_{min} . At sufficiently high energies these processes are dominated by the scattering at impact parameters significantly exceeding the geometrical radius of a target. Therefore at sufficiently large energies initial and final state interactions should become negligible. Thus there exists a practical necessity to evaluate legitimately hadron diffraction into high k_t jets in the ultraperipheral collisions. Moreover in the regime of color opacity (LHC, Tevatron?) proton dissociation into high k_t jets from Coulomb field may dominate in some kinematical region. This is because in this regime the proton diffraction into three jets should be strongly suppressed as compared to the perturbative QCD (pQCD) expectations.

A nucleon (meson, photon) has an appreciable amplitude to be in a state where valence partons are localized in a small transverse area. These configurations are usually referred to as *minimal* Fock space configurations - $|3q\rangle$, $|q\bar{q}\rangle$. A significant amplitude of the decays $\pi(\rho) \rightarrow leptons$ as well as the observed significant cross sections of the processes: $\gamma^* + p \rightarrow V + p$ for $V = \gamma, \rho, \omega, \phi, \psi, \psi'$ [1], and of the process $\pi + A \rightarrow 2jet + A$ [5] provide the experimental evidences for the significant probability of such configurations in a photon, in pseudoscalar and vector mesons. The amplitude of the three quark configuration within proton can be estimated within the QCD

inspired models, and in the long run it can be calculated in the lattice QCD. Note that the knowledge of this amplitude is important for the unambiguous calculation of the proton decay within the Grand Unification models.¹

In [9] V.Gribov has derived a theorem for the radiation by electrically charged particles in two body collisions of an energetic photon but with small momentum relative to the scattering plane. The essence of the theorem is that even at high energies the photon radiation is dominated by the photon emissions from external lines before or after the strong interaction. Later this theorem was used to derive the QCD evolution equation in the Minkowski space [10]. In Ref. [6] the Gribov technique was used to calculate the cross section of the process of pion diffraction of Coulomb field of nucleon(nucleus): $\pi + A \rightarrow jet_1 + jet_2 + A$. It was demonstrated that the amplitude of the pion diffraction into jets off the Coulomb field of nucleus is calculable unambiguously in terms of the high momentum tail of the light cone pion wave function in the leading term in invariant energy, s , α_s and the transverse momentum of the jet, k_t . In this paper we elaborate the analysis of [6] and extend it to calculate the Coulomb dissociation of any hadron (proton, photon) into several jets corresponding either to a minimal Fock configuration, like three jets in the case of the proton, or the multi jet final states containing a gluon jet which originate from the presence of $q\bar{q}g$ components in the meson wave functions. We explain that amplitudes of the discussed processes can be unambiguously calculated in QCD including the effects of the QCD evolution. First, we derive the Williams-Weizecker approximation for the amplitude of ultraperipheral diffraction of a hadron into jets off nucleus Coulomb field and express the amplitude in terms of the field of equivalent transverse photons of the projectile hadron. We give a more detailed explanation of the key observation of [6] that that the dominant contribution into the equivalent photon field of projectile hadron is given by the interaction of transversely polarized photon with the external quark lines within the precision $O(\alpha_s(k_t))$. The physical interpretation of this result is that the radiation of an on-mass-shell photon with a small transverse momentum occurs after the strong interaction and therefore it does not distort the wave function of the hadron.

¹Naively, there appears to be a contradiction between the experimental observation that hadrons consist of an infinite number of partons and the dominance of minimal quark configurations in the light cone hadron wave function of hadrons required for describing hard diffractive processes. The QCD factorization theorem resolves this contradiction by including other partons into generalized parton distributions of nucleon(nucleus) target.

The relative importance of the strong interaction and e.m. mechanisms of dissociation of protons and pions into jets is evaluated. The e.m. mechanism is found to be a small correction in the energy and transverse momentum range where the leading twist effects dominate in the strong amplitude. However if at very high energies the black body limit is reached at a certain range of k_t the e.m. mechanism may become dominant.

2 Evaluation of amplitudes

In this section we evaluate the amplitude of a hadron h diffraction into jets off the Coulomb field of a target of the atomic number A : $h + A \rightarrow jets + A$. At high energies the minimal momentum transfer $-t_{min} R_A^2/3 = m_N^2 \left(\frac{m_{jet}^2 - m_h^2}{\nu} \right)^2 \frac{R_A^2}{3} \ll 1$, where $\nu = \frac{2(p_h p_A)}{A}$, R_A is the e.m. radius of the target, and m_{jet}^2 is the invariant mass² of the produced multijet state. Hence for $t \sim t_{min}$ which dominate in the total cross section, the process is ultraperipheral and there are no initial or final state strong interactions of h or jets with the target.

This amplitude is due to the exchange of a virtual photon of four-momentum q ($q^2 = t$) with the target. The nuclear Primakoff amplitude is then given by

$$\mathcal{M}_h(A) = e^2 \frac{\langle h | J_\mu^{\text{em}} | jets \rangle}{-t} (P_A^i + P_A^f)^\mu \frac{Z}{A} F_A(t) \approx 2e^2 \langle h | J^{\text{em}} \cdot \frac{P_A}{A} | jets \rangle \frac{Z F_A(t)}{-t}, \quad (1)$$

where $F_A(t)$ is the electric form factor of the nucleus. Depending on the quantum numbers of a jet it can be described as a quark, an antiquark, or a gluon jet. A photon can be attached to any charged particle, so a direct calculation of $\langle h | J^{\text{em}} \cdot \frac{P_A}{A} | jets \rangle$ involves a complicated sum of diagrams and would be rather complicated because of the cancellations between the contributions of different diagrams. However we will considerably simplify the calculation by using the conservation of the e.m. current as well as the Sudakov variables. Accordingly, we write :

$$q = \alpha \frac{P_A}{A} + \beta p_h + q_t. \quad (2)$$

Conservation of the four-momentum gives

$$\beta = \frac{q^2}{2(p_h \cdot P_A)}, \quad \alpha = \frac{m_h^2 - m_{jet}^2 - q^2}{\nu} \approx \frac{-m_{jet}^2}{\nu}. \quad (3)$$

Then the conservation of the e.m.current can be rewritten as

$$\langle h|J^{\text{em}} \cdot q|jets\rangle \approx \alpha \langle |J^{\text{em}} \cdot \frac{P_A}{A}|h\rangle + \beta \langle h|J^{\text{em}} \cdot p_h|jets\rangle + \langle h|J^{\text{em}} \cdot q_t|jets\rangle = 0. \quad (4)$$

Using Eq. (3) and keeping only the leading term in μ^2/ν , where μ is the typical mass involved in the considered process, allows us to neglect the β term in Eq. (4) so that

$$\alpha \langle h|J^{\text{em}} \cdot \frac{P_A}{A}|jets\rangle = -\langle h|J^{\text{em}} \cdot q_t|jets\rangle. \quad (5)$$

We want to stress that our formulae are accurate within the leading power of k_t only. So in the matrix element of the e.m. current $\langle h|J_\mu^{\text{em}}|jets\rangle$ we can safely put $q_t = 0$. By definition, the transverse momentum of the pion is zero, so the dominant (in powers of k_t) contribution in Eq. (5) is given by the photon attachments to the external quark lines. Hence in difference from the situation considered by V.Gribov there is no radiation from the initial state. The matrix element is given by ²

$$\langle h|J^{\text{em}} \cdot q_t|jets\rangle = \chi_h(z_i, k_{t,i}) \sum_i \frac{2e_i(\vec{q}_t \cdot \vec{\kappa}_{t,i})}{z_i}. \quad (6)$$

Here e_i is the electric charge of quark in the units of the electric charge of electron. The relative sign of each term can be easily visualized by considering an antiquark as the quark moving in the backward direction. The factor $1/z_i$ arises from the propagator of the interacting quark. This is because the propagator of the interacting quark multiplied by the factor z_i should be included into the definition of the light cone wave function of the projectile hadron. The factor $2k_{t,i}$ in the above formulae arises as a consequence of the commutation of the operator of quark(antiquark) momentum \not{k}_i from the quark propagator with \not{q}_t : $(\not{k} + \not{q})\not{q}_t = 2(k_t \cdot q_t) - \not{q}_t \not{k}$, and neglecting the last term. Indeed the operator \not{k} when acting on the the spinor $u(k)$ leads to a term proportional to $\sqrt{k^2}$, that is the virtuality of the final state (anti)quark which in the leading log approximation is $\ll k_t$ and hence can be neglected. Therefore the effects related to the virtuality of quark(antiquark) in the final state can be neglected in the leading order in $\alpha_s \ln(k_t^2/\Lambda_{QCD}^2)$. The contribution of \not{q} from the propagator of the interacting quark can be neglected as well since it is of the higher order in powers of $1/k_t$. It is easy

²In [6] the factor 2 in the numerator was missing.

to check that in the case when quark (antiquark) transverse momenta are ordered according to the DGLAP approximation the equation derived above coincides with the Gribov formulae for the photon bremsstrahlung.

The generalization of this result to account for all Feynman diagrams having the same powers of ν and k_t is almost trivial. The relative contribution of other diagrams is $\propto k'_t/k_t$ where k'_t is the transverse momentum of quarks in the intermediate state. Therefore the dominant contribution arises from the region of integration: $k_t'^2 \simeq k_t^2$. But within the $\alpha_s \ln k_t^2/\Lambda_{QCD}^2$ approximation $k_t'^2 \ll k_t^2$ so this contribution does not lead to a $\ln k_t^2/\Lambda_{QCD}^2$ term. Thus the above formula is valid within the $\alpha_s \ln k_t^2/\Lambda_{QCD}^2$ approximation when $\alpha_s \ll 1$.

Using Eqs. (5),(1) we obtain the final result:

$$\mathcal{M}_h(A) = \frac{-e^2 \chi_h(z_i, k_{t,i}) Z}{q_t^2 - t_{\min}} F_A(t) \frac{\nu}{m_f^2} \sum_i \frac{2e_i(q_t \cdot k_{t,i})}{z_i} d^{-n/2}(k_{it}^2). \quad (7)$$

Note that in the calculation of the cross section of photon (but not hadron) fragmentation into jets one should multiply square of the above amplitude by the the number of colors. The wave function $\chi_h(z_i, k_t)$ in the above equation describes the high momentum tail of the light cone wave function of a hadron defined as the equal time Bethe-Salpeter wave function which is normalized as in [11]. In the case of the gluon jets with transverse momenta \ll quark transverse momenta $\chi_h(z_i, k_t)$ is calculable in terms of the minimal Fock wave function and formulae for the gluon bremsstrahlung.

In Eq.7 we wrote amplitude in the form which is actually applicable for the description of the amplitude of the diffractive dissociation of any hadron h off the Coulomb field into any number of jets including gluon jets. To include gluon jets production one should substitute in the above formulae the minimal Fock component of the hadron wave function by the component of wave function containing gluons besides of valence quarks. Different projectiles are characterized by the different charges of quarks and by the different number of valence quarks. Thus one should account that $m_f^2 = \sum_i k_{it}^2/z_i$ is different for the system of two or three quarks. Above equation is the basic new result obtained in this paper. We want to draw attention to the curious feature of above expression - in the case of projectile π^+ the amplitude of the process becomes zero when $z_u = 2/3$. This is due to the lack of the transverse dipole strength for such quark-antiquark configuration in π^+ . Similarly for the proton case the amplitude equal to zero when $\frac{2(k_{t,u,1} \cdot q)}{z_{u,1}} + \frac{2(k_{t,u,2} \cdot q)}{z_{u,2}} - \frac{(k_{t,d} \cdot q)}{z_d} = 0$.

We want to draw attention that the renormalization of the wave function

accounts for the cancellation of the infrared divergences [11]. This leads to the additional renormalization factor $d(k_t^2)^{-n}$ in the cross section where $n=2(3)$ for a pion(proton) projectile. cf. [6] (Here $d(k_t^2)$ is the renormalization factor for the quark Green function). A rather straightforward method to avoid such infrared divergences is to use a special light cone gauge in the calculation of the matrix element of the e.m. current: $p_{h,\mu}A_\mu = 0$ [6]. For the proper definition of jets one should sum also over collinear radiation. The high momentum tail of the pion wave function has been calculated in [6].

The Primakoff term for the pion dissociation into two jets has been evaluated also in the paper of D.Ivanov and L.Szymanowski [12]. Similar to [6] they concluded that the e.m. contribution is negligible. However the approximations made in [12] and the results of [12] differ from ours. Our calculation is based on the generalization of the QCD factorization theorem which properly accounts for the conservation of the e.m. current and renormalizability of QCD. On the contrary [12] restricted themselves by the set of diagrams where Coulomb photon may interact with an on-mass-shell $q\bar{q}$ pair in the initial and final states only, which is in variance with the fact that the Coulomb interaction with a bound state is significantly more complicated. They assumed that the $q\bar{q}$ pair in the intermediate states can be put on-mass-shell before the separation of scales and the cancellations between different photon attachments has been taken into account. These approximations have problems with the conservation of the e.m.current and the renormalization group in QCD.

3 Numerical estimates

The ratio of e.m. and strong amplitudes for the hadron diffraction off a nucleus target into high k_t jets is almost independent of the projectile because most of the factors are cancelled out in this ratio. In the case of the pion projectile this ratio has been evaluated in [6] for $q_t^2 = 0.02 GeV^2$, $s = 10^3 GeV^2$, $z = 1/2$, $k_t = 2 GeV$ and $A \sim 200$ to be about $-i0.04$. Since typical q_t are very small, both the fixed target and collider experiments can only measure the cross section integrated over q_t . The strong amplitude is predominantly even under the transposition: $q_t \rightarrow -q_t$ while the e.m. amplitude is odd under the transposition: $q_t \rightarrow -q_t$. Hence averaging over the angles the interference is canceled out.

The relative contribution of the square of e.m. amplitude to the total

cross section of the jet production can be estimated as

$$R \approx \frac{|M(Coulomb)|^2}{|M(strong)|^2} \Big|_{q_t^2=0.02} * (0.02B) \ln \frac{1}{-Bt_{min}},$$

where we used an exponential fit to the nuclear electric form factor: $F_{em}(t) = \exp Bt$, where $B = \frac{R_A^2}{6}$ and $R_A = A^{1/3} 1.1 Fm$.

For the kinematics of the FNAL experiment we get for the ratio of e.m. and strong contributions to the diffractive yield of jets for heavy nuclei a value of about $5 \cdot 10^{-3}$. For LHC two factors work in opposite directions - the e.m. contribution is enhanced by a factor of k_t^2 but suppressed due to a fast increase of the gluon density with decrease of x , and increase of k_t . Taking, for example, $s \approx 10^8 GeV$ and $k_t = 10 GeV$ we find approximately the same ratio for e.m. and strong contributions.

For the case of proton-nucleus scattering we expect similar ratios, though a more quantitative analysis would require a more detailed treatment of the strong amplitude.

In the estimates made in the paper we used the energy dependence of the pQCD amplitude as given by LO pQCD. At the same time there exists serious reasons to anticipate black body regime for the pQCD physics in pA collisions at LHC, Tevatron for jet transverse momenta $k_t \leq k_{t0}$ [8, 13]. In this case pQCD amplitude will be strongly suppressed by the shadowing effects since no inelastic diffraction is possible in the black body limit anywhere but near the nuclear edge. This leads to $\sigma \propto \frac{A^{2/3}}{R_A^2} \propto const(A)$, while the A-dependence of the Coulomb contribution is not changed. So if the interaction becomes black at some k_t scale e.m. contribution starts to win.

One practical possibility to investigate the e.m. mechanism and to measure the photon wave function would be to study at high-energy electron positron colliders the processes: $e^+e^- \rightarrow e^+ + two\ jets + e^-$,³ and $e^+e^- \rightarrow e^+ + three\ jets + e^-$.

Formulae derived in the paper are equally applicable to the QED processes like diffractive photoproduction of high k_t lepton pairs off the nuclear field which has been considered firstly in [14], the diffractive fragmentation of positronium into high k_t electron-positron pairs, etc. In these cases electric charges of constituents are equal but have opposite signs. As a result, the basic equation can be rewritten in terms of the derivative over k_t of the light-cone wave function of an electrically neutral projectile. This result has been

³S.Brodsky, A comment at the Bad Honnef workshop, July 2002

anticipated in [15]. In the case of charged hadron fragmentation into quark jets formulae have more complicated structure.

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